TOWARDS LEVELING UP IN THE EUCLIDEAN ALGORITHM: EVIDENCE FROM THIRD YEAR STUDENTS

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Abstract

The Euclidean algorithm is a fundamental mathematical procedure for finding the greatest common divisor of two integers. Yet research shows students often struggle to apply the algorithm accurately and efficiently. This research investigates the leveling-up plan in Number Theory, focusing on the Euclidean Algorithm. The study employs a descriptive research method and utilizes a purposive sampling technique. The researcher-designed instrument demonstrates high reliability, with a Cronbach's Alpha coefficient of 0.897. The key objective includes assessing and evaluating the extent of the students' performance in the Euclidean Algorithm. Results indicate a statistically significant difference to a great extent in the students' performance in Number Theory, focusing on the Euclidean Algorithm. This discovery is significant because the performance of the students will be improved in problem-solving using the Euclidean Algorithm.

In light of these findings, recommendations are proposed. It is suggested that students should be trained to solve problems independently to improve their ability to overcome difficulties and master the Euclidean algorithm. Moreover, teachers should intervene to educate students on using mental processes properly and recognize student errors and misconceptions in order to develop effective, modern, and inclusive math instruction on the Euclidean algorithm. Additionally, future research could further explore instructional techniques targeting specific student misconceptions, as well as factors impacting student performance with the Euclidean algorithm. Finally, full implementation of Project LEARN: Leveling up plan towards Euclidean Algorithm Recursive Numeric to help students gain conceptual understanding, procedural skills, and computational fluency with the Euclidean algorithm.

Keywords: *Education; Euclidean Algorithm; Mathematics; Number Theory; Teaching and Learning*

Introduction

Mathematics education in the twenty-first century involves confronting novel real-world problems, cultivating creative thinking skills, and fostering productive learning practices. Many educators have discovered the value of metacognition in their efforts to innovate teaching and learning in order to prepare a new generation for the challenges of this new era. The metacognitive ability of the problem solver varies and can affect the problem-solving performance in Mathematics of the students. Based on the study of Albay (2019), results established evidence that the problem-solving approach when applied to classroom instruction can significantly improve understanding and performance in mathematics and can also promote a favorable attitude towards the subject, hence indicating the effectiveness of the approach in teaching mathematics. This discovery is significant because the students will become more independent in discovering and finding ways to solve problems in mathematics, specifically the problem-solving in the Euclidean Algorithm. As cited in Aguhayon (2023, in the PISA 2018 International Report, Filipino students' average score in mathematical literacy was 353 points, significantly lower than the Organization for Economic Cooperation and Development (OECD) average of 489 points, indicating a below Level 1 proficiency (OECD, 2019). The Philippines also scored 297 in math in the 2019 Trends in International Mathematics and Science Study (TIMSS) by the International Association for the Evaluation of Educational Achievement (Mullis et al., 2019). The ability to engage in metacognitive activity is an essential component of solving mathematical problems. To solve mathematical problems, one must first analyze the problem at hand, then plan out a strategy that can be applied to deal with the problem, then put that strategy into action, and finally verify that the steps that were taken to solve the problem were correct.

It is necessary to take this variable into consideration to create more effective learning strategies, especially in the field of mathematics. To fully assess the significance of a person's learning capability and improve their problem-solving skill in mathematics, it is necessary to understand the metacognition of the learners towards leveling up in Mathematics in the Modern World, to analyze, evaluate, and create. It is now timely to investigate the relationship between the metacognition level of the students and their achievement in leveling up their problem-solving skills in Mathematics in the Modern World. The researcher is eager to find out those potentials for this research study to become a related study and a reference someday. Solving problems is a crucial part of learning mathematics. Mathematics is a vital subject due to its many applications in which it may benefit society. Another quality of problem-based learning is that it may be used to motivate students to generalize rules and concepts, a skill that is crucial in mathematics. Solving mathematics problems requires analysis of the given problem, planning the strategy to be used to solve the problem, undertaking the planned strategy, and checking whether the steps that have been done are correct. Therefore, metacognition is necessary for the successful solving of mathematical problems. This paper analyzes that the higher metacognition that students have, the better mathematical problem-solving that they can do. (Izzati and Mahmudi, 2018)

The research attempted to determine the extent of achievement in the Euclidean Algorithm of the students at Cavite State University-CCAT Campus towards the leveling up plan. Multiple studies found a strong positive link between students' mathematics performance/competence and their overall academic achievement. Students who were excellent in math tended to excel across subjects. Effective study strategies will predict better math performance in students. Some research indicates that math competence promotes academic success, while negative attitudes, poor teaching, and lack of support undermine outcomes. Targeted, motivational interventions and teacher training can help at-risk math students succeed.

Problem-solving is a process. It is a skill requiring a more difficult or complicated level of cognitive functioning than the level required for basic computation. The ability to solve problems does not develop automatically as students or children master the computation skills, and solving problems is different from doing an exercise in mathematics. In addition, one important feature of mathematical problem-solving ability is the ability to convey concepts and procedures acquired for one problem in one setting to new problems in a new setting. This ability is particularly useful when new problems that arise have similar elements to previous problems that have been solved.

Methods

Descriptive method was used by the researchers to simplify complex situations and information's making it easier to understand and analyze. It is also designed to describe the nature of the study's particular situation as well as explore causes of particular phenomena. The researcher conducted the study at Cavite State University-CCAT Campus at Rosario, Cavite that offers nine (9) degree programs. Purposive sampling technique was used to select a specific group of individuals or units for analysis. This study was limited to the selected five (5) third year Bachelor of Secondary Education (BSE) major in Mathematics students of the Campus that deals primarily with the students' level in problem solving in Number Theory focusing on Euclidean Algorithm.

A reliability test was carried out with the help of Cronbach Alpha to assess the reliability of the research instrument. It showed that the reliability is 0.897 that is interpreted as very high reliability. The instrument was given to the selected third year BSE major in Mathematics of the Campus. They were given problem solving questions focusing on Euclidean Algorithm. The respondents responded with a 5-point Likert Scale to provide numerical values ranging from 5 (Outstanding) through 1(Needs Improvement). t-test was used, specifically the paired sample t-test, to determine the significant difference on the extent of the performance of the students in Number Theory focusing on Euclidean Algorithm. T-test or t- distribution differs appreciably from the normal form for smaller sample size. Correspondingly, the theoretical sampling distribution of t-deviates from the normal probability distribution depending on the sample size. Also to check if there is a statistically significant difference between the two related or paired measurements.

Results

The students performed to a very great extent on Step 1, Identifying the Problem, in solving the Euclidean Algorithm, with an average of 4.40 and a standard deviation of 0.447, which is homogeneous, showing that the data are close to one another. In the paired sample tested, it was found that on Step 1, there is a statistically significant mean difference between the students' scores and the teacher's scores in the performance of the students. The results of the paired sample t-test showed that the students' scores and the teacher's scores were statistically significantly different, $t(4) = 4.000$, $p < 0.05$. Therefore, we reject the null hypothesis. This provides sufficient evidence to suggest that students' scores, Identifying the Problem, are significantly different from the teacher's scores on the performance of the students. Based on the result, the students' performance to a very great extent shows they can recognize and understand Euclidean Algorithm word problems that will help them set up and solve these problems.

Table 1. Paired-Sample Test on the Extent of the Performance of the Students on the Identification of the Problem in the Euclidean Algorithm.

Steps	x			$\tilde{x}$ **Diff**	**SD**	**Descriptive Level**	**t-Value**	**Sig. (2-tailed)**
	A	**B**	**Ave**					
Step 1: Identifying the Problem GCD (A, B)	4.80	4.00	4.40	0.80	0.447	To a Very High Extent	4.000	0.016

Legend: A-Students Rating, B-Teachers Rating,

On Step 2, which is the Assumptions on Euclidean Algorithm, the students performed to a neutral extent, with an average of 3.20 with a standard deviation of 0.837, which means most scores tend to fall within about 0.837 points of the mean in either direction. Using the paired sample t-test, it was revealed that there is a statistically significant mean difference between the students' scores and the teacher's scores in the performance of the students. The results of the paired sample t-test rejected the null hypothesis that showed that the students' score and the teacher's score were statistically significantly different, $t(4) = 3.207$, $p < 0.05$. This provides sufficient evidence to suggest that students' scores in terms of the assumptions on Euclidean Algorithm are significantly different from the teacher's score on the performance of the students. Struggling with assumptions suggests more examples, and practicing different problems could help. Teachers should spend more time explaining the logic behind Euclidean Algorithm assumptions, not just memorizing them, to ensure students understand the assumptions.

Table 2. Paired-Sample Test on the Extent of the Performance of the Students on the Assumptions in the Euclidean Algorithm.

Steps	x			$\tilde{x}$ Diff	SD	Descriptive Level	t-Value	Sig. (2-tailed)
	A	B	Ave					
Step 2: Assumptions If A=0, GCD (A, B) = B. If B=0, GCD (A, B) = A.	3.80	2.60	3.20	1.20	0.837	To a Neutral Extent	3.207	0.033

Legend: A-Students Rating, B-Teachers Rating,

The students performed well to a great extent on Step 3, solving problems by applying the Euclidean algorithm. The average mean score was 4.10, with a homogeneous standard deviation indicating that the data were reasonably close to each other and that the majority of scores tended to fall within 0.707 points of the mean in either direction. Applying the paired sample t-test, it was discovered that there is a statistically significant mean difference between the students' and the teacher's scores on the students' performance. The paired sample t-test revealed a significant difference between the students' and teachers' scores, $t(4) = 3.162$, $p < 0.05$. Thus, we reject the null hypothesis. This provides substantial data to show that students' scores, solving problems using the Euclidean method, deviate considerably from the teacher's score on students' performance. Based on the high result, it showed that the students have mastered the way of the Euclidean Algorithm to solve the problem, which gives them a solid basis for moving on to more difficult problems and procedures.

Table 3. Paired-Sample Test on the Extent of the Performance of the Students on the Application of the Algorithm on the Problem in the Euclidean Algorithm.

Steps	x			$\tilde{x}$ Diff	SD	Descriptive Level	t-Value	Sig. (2-tailed)
	A	B	Ave					
Step 3: Solving the Problem Applying the Algorithm A = BQ + R	4.60	3.60	4.10	1.00	0.707	To a High Extent	3.162	0.034

Legend: A-Students Rating, B-Teachers Rating,

On the last step, Step 4, which is the conclusion of the Euclidean Algorithm, the students performed to a great extent with an average of 4.20 with a standard deviation that is homogeneous, showing that the data are compressed, and that most scores tend to fall within about 0.447 points of the mean in either direction. Using the paired sample t-test, it was revealed that there is a statistically significant mean difference between the students' scores and the teacher's scores in the performance of the students. The results of the paired sample t-test rejected the null hypothesis that showed that the students' score and the teacher's score were statistically significantly different, $t(4) = 4.000$, $p < 0.05$. This provides sufficient evidence to suggest that students' scores in Step 4 are significantly different from the teacher's score on the performance of the students. This implies that students gain confidence and strengthen their conceptual knowledge of the problem-solving process on the Euclidean Algorithm.

Table 4. Paired-Sample Test on the Extent of the Performance of the Students on the Conclusion of the Euclidean Algorithm.

Steps	x			$\tilde{x}$ **Diff**	**SD**	**Descriptive Level**	**t-Value**	**Sig. (2-tailed)**
	A	**B**	**Ave**					
Step 4: Conclusion GCD (A, B)	4.60	3.80	4.20	0.80	0.447	To a High Extent	4.000	0.016

Legend: A-Students Rating, B-Teachers Rating,

Solving the mathematical problems needs analysis of the provided problem, preparing the strategy to be utilized to solve the problem, performing the planned method, and assessing whether the actions that have been taken are accurate. A good mathematical problem solver must thoroughly follow the steps in solving the Euclidean Algorithm. The performance of the students will be improved in problem solving in Number Theory, specifically on the Euclidean Algorithm, when they become metacognitively aware of their own learning. Students will be more encouraged to respond to problems in mathematics. Additionally, it will help them become responsible and self-sufficient students. Moreover, students should follow the algorithmic steps in solving the Euclidean Algorithm to make it mathematically valid, efficient, consistent, and easy to understand. Furthermore, strictly following the Euclidean algorithm guarantees an accurate, efficient GCD calculation in a clear, methodical, and mathematically valid way. This makes the Euclidean algorithm reliable, teachable, and instructive for one of the most fundamental concepts in number theory.

Based on the results, the proposed program is called "Project LEARN: Leveling up plan towards Euclidean Algorithm Recursive Numeric". It is a proposed plan that will enhance the students' performance in Euclidean Algorithm, a method for finding the greatest common divisor of two or more numbers, to cope with the recursive numerical processes more efficiently or effectively. It aimed for the students to understand more of the mathematical basis and logic behind the algorithm; to master the steps and assumptions involved in applying the algorithm; and to develop proficiency in performing the calculations manually. In terms of the teaching methodologies to be used by the teachers in teaching the Euclidean Algorithm, it is recommended to utilize the Project LEARN.

Discussion

The null hypothesis was rejected that provides evidence that the mean performance of the students using the students' scores in Number Theory focusing on Euclidean Algorithm is significantly different to the high extent from the teachers' scores. The students will become more independent in discovering and finding ways to solve problems in mathematics, specifically the problem solving in the Euclidean Algorithm. Based on the study of Albay (2019), results established evidence that the problem-solving approach when applied to classroom instruction can significantly improve understanding and performance in mathematics and can also promote a favorable attitude towards the subject, hence indicating the effectiveness of the approach in teaching mathematics.

Considering the findings and conclusion of this study, it is suggested that training students to solve problems on their own is part of teaching them to think. This will improve their ability to overcome difficulties on their path to mastering a foundational mathematical procedure in the Euclidean Algorithm. Also, the teachers shall make interventions to recognize student errors and misconceptions in the Euclidean algorithm to improve teaching approaches by placing more emphasis on the assumptions, not just memorizing, to ensure students' understanding of the assumptions. Finally, it is suggested to fully implement the Project LEARN, which will make the students gain conceptual understanding of the Euclidean algorithm, procedurally learn how to apply it, and develop skills for accurate manual implementation.

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